NAME:
Math 150 Exam 3

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. True or False?
   a) If f and g are continuous on [a, b], then

   $$\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$$ [2 pts]

   b) $\int_{-1}^1 \left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2}\right)dx = 0$. [2 pts]

   c) All continuous functions have antiderivatives. [2 pts]

   d) If $\int_0^1 f(x)dx = 0$, then $f(x) = 0$ for $0 \le x \le 1$ [2 pts]

   e) $\int_0^3 e^{x^2}dx = \int_0^5 e^{x^2}dx + \int_5^3 e^{x^2}dx$ [2 pts]

2. Evaluate $\int_0^1 \frac{d}{dx}\left(e^{\tan^{-1}x}\right)dx$ [10 pts]

3.   Evaluate $\int_0^3 |x^2 - 4| dx$ [10 pts]

4.   Evaluate $\int_0^1 v^2 \cos(v^3) dv$ [10 pts]

5.   Evaluate $\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx$ [10 pts]

6.   Find the derivative of the function $f(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\,dt$ [10 pts]

7. Express the limit $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{4}{n}\left(2+\dfrac{4}{n}\right)\ln\left(1+\left(2+\dfrac{4}{n}\right)^2\right)$ as a definite integral. Do

   not evaluate. [10 pts]

8. Use the properties of integrals to verify the inequality $\displaystyle\int_0^1 x^2 \sin\sqrt{x}\,dx \leq \dfrac{1}{3}$

   [10 pts]

9.   Let $f(x) = 2x + x^2$. Use the right-hand method to compute the area $\int_{-1}^{1} f(x)dx$. [Hint: to compute R_n, you will need the formula $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$]   [10 pts]

10.   Evaluate $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$ [10 pts]

Extra-Credit

11.   Evaluate $\lim_{n \to \infty} \dfrac{\pi}{2n} \left[Cos\left(\dfrac{\pi}{2n}\right) + Cos\left(\dfrac{2\pi}{2n}\right) + Cos\left(\dfrac{3\pi}{2n}\right) + ... + Cos\left(\dfrac{n\pi}{2n}\right) \right]$

[10 pts]